Cellular Automata Model Of Emergent Collective Bi-Directional Pedestrian Dynamics

V. J. Blue[1]∗ and J. L. Adler[2]‡

[1] New York State Department of Transportation, Poughkeepsie, NY 12603
[2] Department of Civil Engineering, Rensselaer Polytechnic Institute Troy, NY 12180

ABSTRACT

This paper describes the application of an Artificial Life cellular automata (CA) microsimulation to model the emergent collective behavior of bi-directional pedestrian flows. Since pedestrian flow is inherently complex, even more so than vehicular flow, previous CA models developed for vehicle flow are not directly applicable. It is shown that a relatively small rule set is capable of effectively capturing the collective behaviors of pedestrians who are autonomous at the micro-level. The model provides for simulating three modes of bi-directional pedestrian flow: (a) flows in directionally separated lanes, (b) interspersed flow, and (c) dynamic multi-lane flow. The emergent behavior that arises from the model is consistent with well-established fundamental properties of pedestrian flows.

INTRODUCTION

Cellular Automata (CA) microsimulation is an effective technique for modeling complex emergent collective behavior that is characterized as an Artificial Life approach to simulation modeling (Adami 1998; Levy 1992). CA is named after the principle of *automata* (entities) occupying *cells* according to localized neighborhood rules of occupancy. The CA local rules prescribe the behavior of each automaton creating an approximation of actual individual behavior. Emergent collective behavior is an outgrowth of the interaction of the microsimulation rule set over local neighborhoods.

Traditional simulation models apply equations rather than behavioral rules, but CA behavior-based cellular changes of state determine the emergent results. The self-organization in the collective behavior of Artificial Life modeling stems from decentralized sources of decision making, such as ant colonies, flocks of birds, and vehicles. CA pedestrian simulation, as used here, is a parallel, distributed, bottom-up approach (see Resnick, 1994 for example). By "designing" the CA-based pedestrian from the bottom-up at the interface with one another, higher-level functions, like route selection and trip behavior, can be added later without fundamentally changing the inter-pedestrian dynamics.

CA models are attractive for a number of reasons. The CA interactions of the pedestrians are based on intuitively understandable behavioral rules. They are easily implemented on digital computers, and compared to difference equation-based microsimulation models, run exceedingly fast. CA models function as discrete idealizations of the partial differential equations that describe fluid flows and allow simulation of flows and interactions that are otherwise intractable (Wolfram, 1994). Only the local rules and the sequencing of their use are coded, leaving the many autonomous interactions on the cell matrix to create the emergent macroscopic results. As a result it has been observed in CA simulations that very simple models are capable of capturing essential system features of extraordinary complexity (Bak, 1996).

Over the past several years, researchers have demonstrated the applicability of cellular automata (CA) microsimulation to car-following and vehicular flows. These CA models have included traffic within a single-lane (Nagel and Schreckenberg, 1992), two-lane flow with passing (Rickert, et al., 1995), bi-directional two-lane flow with passing (Simon and Gutowitz, 1998), and network-level vehicle flows in the TRANSIMS model (Nagel, Barrett, and Rickert, 1996). CA traffic models have been shown to provide a good approximation of complex traffic flow patterns over a range of densities (Nagel and Rasmussen, 1994; Paczuski and Nagel, 1995; and Nagel, 1996) including the formation of shock waves in traffic jams.

Though the field of traffic flow modeling is well established, researchers have found the task of modeling pedestrian flows to be somewhat daunting. In several ways, pedestrian movements are more complex than vehicle flows. Pedestrian corridors may have several openings and support movement in several directions. Pedestrian walkways are not regulated as roadways are. Unlike roadways where vehicle flow is separated by direction, bi-directional walkways are the norm rather than the exception. For the most part pedestrian flows are not channeled by direction, leaving pedestrians free to vary speed and occupy any part of a walkway. Pedestrians can form lanes dynamically as the authors have observed at Grand Central Station and on streets in New York City.

∗ vjblue@gw.dot.state.ny.us
‡ adlerj@rpi.edu

Pedestrians are capable of changing speed more quickly, in one second accelerating to full speed from a standstill or braking to a stop from full speed. Also, since safety and crash avoidance are less of a concern to pedestrians, sidestepping, slight bumping, nudging and exchanging places are often a part of walking through crowded corridors.

Over the past thirty years, researchers have developed several approaches to model pedestrian flows (e.g., Fruin, 1971; AlGhadi and Mahmassani, 1991; Lovas, 1994). Gipps and Marksjo (1985) developed a CA-like model that focused on the use of reverse gravity-based rules to move pedestrians over a grid of hexagonal cells. Helbing and Molnar (1995) advanced a social force model of pedestrian dynamics that captures some properties of bi-directional pedestrian flows, including the formation of dynamic multiple lanes, but is burdened with high computational overhead from floating point calculations. Hoogendoorn and Bovy (2000) have very recently built a gas-kinetic model of pedestrians. Work is underway in the STREETS model (Shelhorn et al., 1999) to create agents with the SWARM simulation system (Langton et al., 2000) that can navigate from behavioral attributes along intended routes and at the same institute a similar path finding approach over spatial systems is underway (Batty, Jiang, and Thurstain-Goodwin, 1998).

The CA model of pedestrian walkways presented here has several advantages, including an intuitively appealing emulation of pedestrian behavior and reliance on integer arithmetic for fast computation. Blue and Adler (1998) demonstrated that unidirectional pedestrian flows emerged from CA simulation experiments that correspond to the fundamental parameters published in a chapter dedicated to pedestrian characteristics in the recent edition of the U.S. Transportation Research Board's Highway Capacity Manual (1994). They further demonstrated the basic framework for the bi-directional pedestrian flow model and examined flows under different exchange probabilities with freer lane changing (Blue and Adler 1999a, 1999b, 1999c).

This bi-directional pedestrian modeling effort is distinctly different from the bi-directional vehicle model developed by Simon and Gutowitz (1998). The bi-directional vehicular flow model focuses on modeling acceleration and passing movements within the framework of two lanes of opposing flow. Due to high speeds of vehicles and the seriousness of collisions, vehicle movements require a more global view of a roadway segment. However, as roadway density increases, the level of vehicular activity (such as lane changing and acceleration) decreases significantly. Pedestrian flow, on the other hand, occurs at lower speeds and collisions are less catastrophic. As a result, pedestrians can adopt a more myopic view of their surroundings. Unlike vehicular flow, as density increases on pedestrian walkways, there can be substantial activity as lane changing, exchanges, and speed variations occur frequently, including the dynamic formation of lanes of various widths.

This paper discusses the application of the bi-directional pedestrian flow model to three modes: (a) separated directional flows (essentially two unidirectional flows), (b) interspersed directional flows, and (c) dynamic multi-lane (DML) flows as important cases that can be treated with the model. Also examined is the importance of lane changing (sidestepping) and place exchange in the CA pedestrian model. Simulation results are presented to demonstrate the CA method's ability to capture fundamental properties of pedestrian movements.

CELLULAR AUTOMATA RULE SET

By incorporating a rule set that eliminates anything but critical behavioral factors, the model facilitates a clear understanding of the underlying fundamental dynamics. There are three fundamental elements of pedestrian movements that a bi-directional microscopic model should account for: side stepping (lane changing), forward movement (braking, acceleration), and conflict mitigation (deadlock avoidance). The basic rule set for the model was developed around these three elements and is designed to work within a framework of parallel updates. As used by Rickert et al. (1995) and Simon and Gutowitz (1998) lane assignment and forward motions change the positions of all pedestrians in two parallel update stages in each time step. Parallel updates avoid succession interdependencies encountered in sequential updates by determining all the new positions before anyone moves. All the entities are then repositioned together. Only the pedestrians in the immediate neighborhood affect the movement of a pedestrian, which, though myopic, is relatively realistic. Each pedestrian is randomly assigned a desired speed of 2, 3, or 4 cells per time step or v_max from a normal distribution of walker speeds (Blue and Adler, 1998). The rule set is presented in Table 1.

In the first parallel update stage, a set of lane changing rules is applied to each pedestrian on a lattice of square cells to determine the next lane of each pedestrian based on current conditions. The lane that best promotes forward movement is chosen from the local decision neighborhood, consisting of the left, same, and right lanes. Once sidesteps are found for everyone, all the pedestrians are moved to the new cells. In the second parallel update, a set of forward movement rules is applied to each pedestrian. The allowable movement (and thus the speed) of each pedestrian is based on the pedestrian's desired speed and the available gap ahead as constrained by the pedestrian in its current position directly ahead. Once speeds are found for everyone, all the pedestrians "hop" forward to new cells.

Pedestrians can change lanes only when an adjacent cell is available. A random number is drawn to designate the lane as free to this pedestrian or to the pedestrian two cells away. If an adjacent lane is free, then lane change is determined by the maximum gap ahead. The base case is interspersed (ISP) bi-directional flow in which pedestrians and exchange places but do not form lanes.

Lane change (parallel update 1):

(1) Eliminate conflicts: two walkers that are laterally adjacent may not sidestep into one another
  (a) an empty cell between two walkers is available to one of them with 50/50 random assignment

(2) Identify gaps: same lane or adjacent (left or right) lane is chosen that best advances forward movement up to *v_max* according to the gap computation subsection* that follows the step forward update
  (a) For dynamic multiple lanes (DML):
    (i) step out of lane of a walker from opposite direction by assigning gap = 0 if within 8 cells
    (ii) step behind a same direction walker when avoiding an opposite direction walker by choosing any available lane with gap_same = 1 when gap = 1
  (b) ties of equal maximum gaps ahead are resolved according to:
    (i) 2-way tie between the adjacent lanes: 50/50 random assignment
    (ii) 2-way tie between current lane and single adjacent lane: stay in lane
    (iii) 3-way tie: stay in lane

(3) Move: each pedestrian p_n is moved 0, +1, or –1 lateral sidesteps after (1)-(3) is completed

Step forward (parallel update 2):

(1) Update velocity: Let $v(p_n)$ = gap where gap is from gap computation subsection below*

(2) Exchanges: IF gap = 0 or 1 AND gap = gap_opp (cell occupied by an opposing pedestrian) THEN with probability *p_exchg* $v(p_n)$ = gap + 1 ELSE $v(p_n) = 0$

(3) Move: each pedestrian p_n is moved $v(p_n)$ cells forward on the lattice.

Subprocedure: Gap Computation

(1) Same direction: Look ahead a max of 8 cells (8 = 2 * largest *v_max*) IF occupied cell found with same direction THEN set gap_same to number of cells between entities ELSE gap_same = 8

(2) Opposite direction: IF occupied cell found with opposite direction THEN set gap_opp to INT (0.5 * number of cells between entities) ELSE gap_opp = 4

(3) Assign gap = MIN (gap_same, gap_opp, *v_max*)

Table 1. Rule Set

At this point in the lane-change parallel update (Lane Change Rule 2a) dynamic multiple lane (DML) flows will result by assigning a forward gap of 0 if encountering an opposing entity ahead. DML formations are further enhanced when a pedestrian can step behind a same direction walker when avoiding an oncoming pedestrian. This realistic behavioral adjustment helps the pedestrians to move into a same-direction flow lane.

Figure 1 illustrates this emergent DML pattern after 100 seconds of simulation time. In this figure, pedestrians move east-west with the darker cells representing eastbound pedestrians, lighter gray representing westbound pedestrians, and white cells being void. Although the pedestrians were randomly placed at the start of the simulation, after 100 seconds the pedestrians have found their way into same-direction lanes.

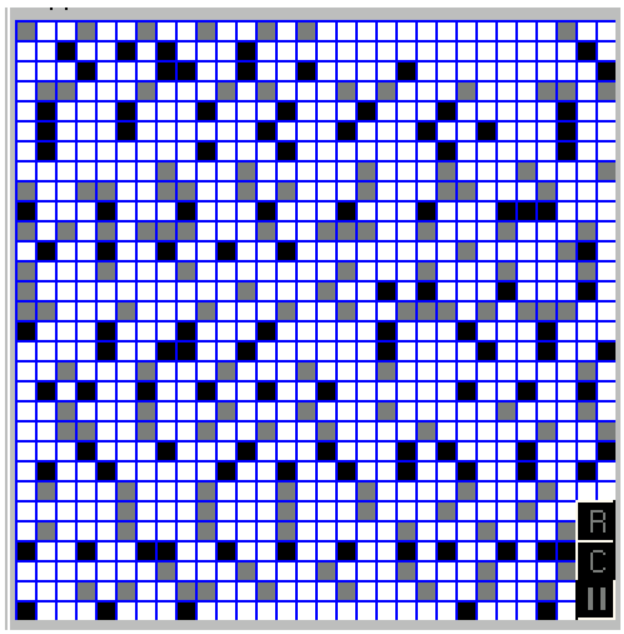

Figure 1. Emerging Dynamic Multiple Lanes

If the maximum gap size is common to two or more lanes, ties are broken to make lane assignments. In this rule set if the current lane is maintained as much as possible, speeds increase over allowing for some lane changes. Evidently, optional lane changes introduce blocks to those who would clearly benefit by changing lanes. Finally, all the pedestrians are moved together into their new lanes.

The second parallel update stage determines the forward movement of the pedestrians. The gap ahead is determined first. The available gap ahead depends on the direction of flow of the next person downstream. From the gap calculation, if the pedestrian ahead is going in the same direction, the new velocity of the follower is the minimum of the desired velocity (*v_max*) and the available gap ahead. If the pedestrian immediately ahead is going in the opposite direction and within the local neighborhood that both pedestrians could move at maximum speed (i.e., 8 cells is the maximum – 4 in each direction), then the updated velocity is the minimum of *v_max* and moving halfway forward. Moving halfway forward guards against collisions and hopping over one another.

For opposing pedestrians, place exchange guards against deadlocks by emulating what people actually do. Under constrained conditions opposing pedestrians may slip by one another. People are somewhat elastic and certainly not perfectly square and can thus exchange places. In actuality, temporary standoffs may occur when people guess which way to step past one another. Thus, the simulation contains a probability of a temporary standoff between closely opposing walkers. With probability *p_exchg* closely opposing pedestrians exchange places in the time step. The opposing entities each move the same number of cells, which is 0, 1, or 2 cells. Finally, the pedestrians go forward based on the gaps.

SIMULATION EXPERIMENTS

Since walking speed varies among pedestrians, a distribution of walking speeds is needed. For this simulation effort, three walking groups were used:

(a) *Fast Walkers* -- maximum speed of 4 cells per time step (about 1.8 m per time step).

(b) *Standard Walkers* – maximum rate of 3 cells per time step (about 1.3 m per time step)

(c) *Slow Walkers* – maximum rate of 2 cells/time step (about 0.85 m per time step)

A 5% fast; 90% standard; 5% slow (5:90:5) distribution of walkers were used to represent the pedestrian population. This distribution had the best realization of the fundamental diagram compared with other distributions in the single-direction case (Blue and Adler, 1998). The distribution also is consistent within ranges of speed and standard deviations used by others (Lovas, 1994).

The pedestrian walkway is modeled as a circular lattice of width W and length G (a rectangular grid that wraps around at the narrow ends). Each cell in the lattice is denoted $L(i, j)$ where $1 \le i \le W$ and $1 \le j \le G$. Pedestrian densities are predetermined at the start of the simulation and remain constant throughout each run. At the start of each simulation, a density d, where $0.05 \le d < 1.0$, is generated and $N = INT(d*W*G)$ pedestrians are created and assigned randomly to the lattice. The circular lattice enables the set of pedestrians to interact at constant density and constant space allowance while maintaining strict conservation of flow. Cells in the lattice are considered square at 0.457 meters per side. This cell size, 0.457m on a side, is scaled according to minimal requirements for personal space as described in the Highway Capacity Manual (1994). The scale is also used to generate the speed-flow-density relationships that emerge. A 50x50 lattice of cells is used in the simulation experiments.

One second is the duration of each time step. Each simulation is 1,000 time steps with the first 100 time steps discarded to initiate the simulation and the latter 900 (15 minutes) used to generate performance statistics. The lattice loops back upon itself, allowing the pedestrians to continuously walk on a circular track.

Each set of experiments included runs at 19 densities ranging from 0.05 to 0.95 percent occupancy in increments of 0.05. For statistical accuracy, ten replications at each density level were run and the fundamental parameters were computed as the average over these replications. The emergent fundamental profile of the model is a map of the relationships between speed and flow over the range of densities.

RESULTS

The CA model yields flows that should be expected. As discussed earlier and depicted in Figure 1, dynamic multiple lane formation is generated by the bi-directional model. Figure 2 demonstrates the ability of the model to generate mode locking, a marching effect emerges from unidirectional flow that is very efficient, especially at low density. Mode locking is often seen in complex, self-organizing systems (see Schroeder 1991). This figure depicts the simulation applied to a uni-directional walkway in which all pedestrians are moving eastbound. Several rows in the walkway appear to be almost identically populated with pedestrians, illustrating the mode locking phenomenon.

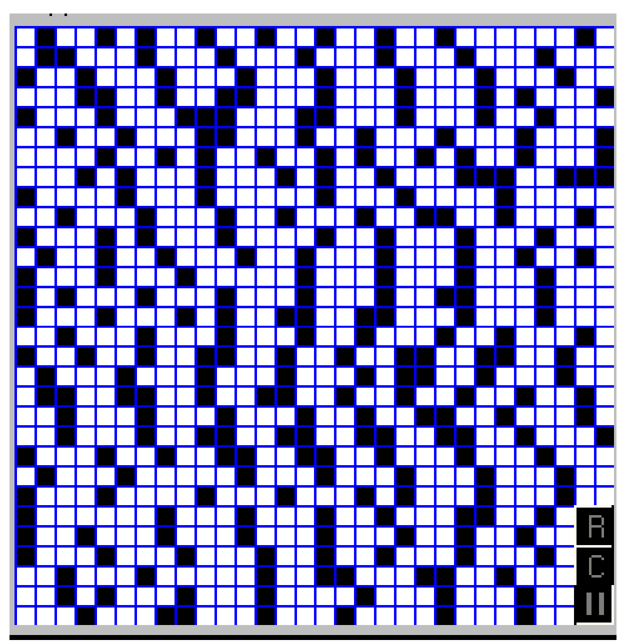

Figure 2. Mode Locking

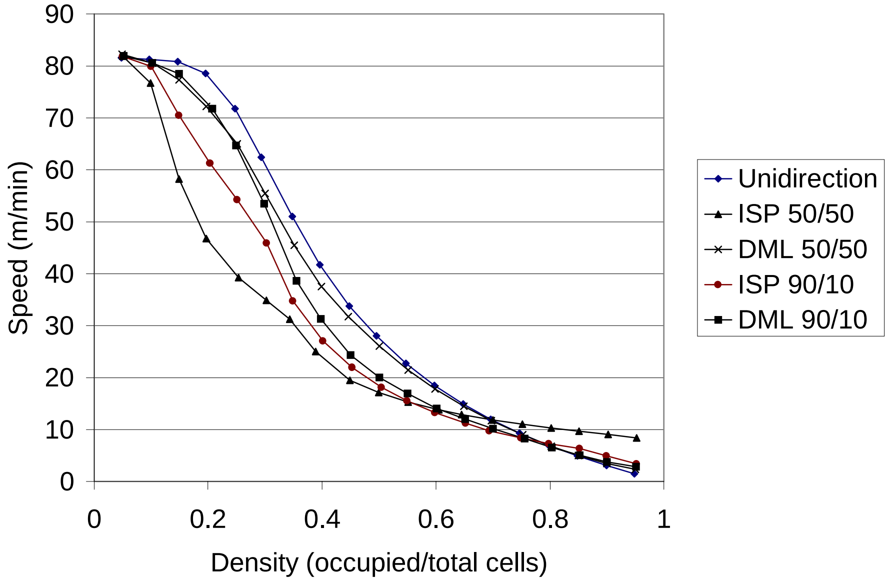

Figure 3. Speed vs density;p_exchg = 0.5

The results of simulations under conditions of (a) unidirectional flow, (b) interspersed bi-directional balanced flows (50-50), (c) dynamic multiple lane bi-directional balanced flows (50-50), (d) interspersed bi-directional unbalanced flows (90-10), (c) dynamic multiple lane bi-directional unbalanced flows (90-10).

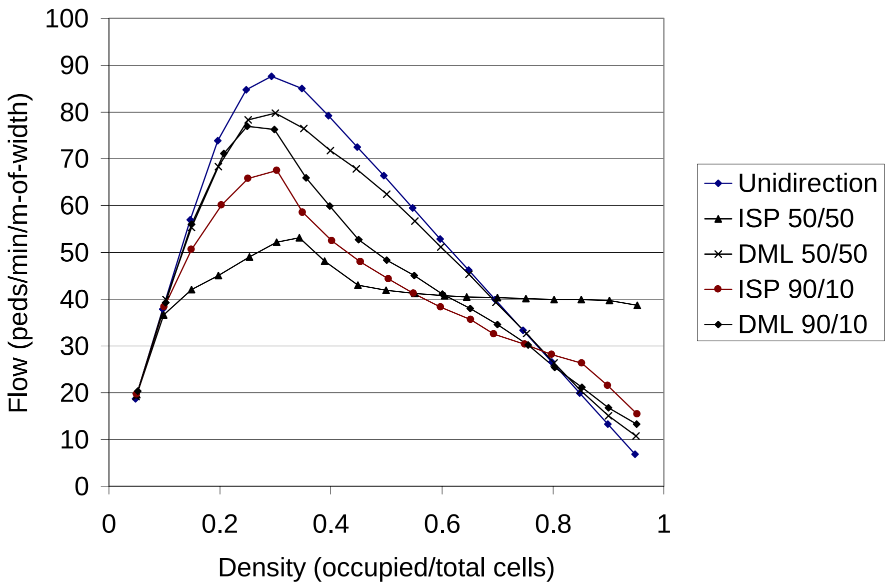

Figure 4. Flow vs density; p_exchg = 0.5

The results of simulations under conditions of (a) unidirectional flow, (b) interspersed bi-directional balanced flows (50-50), (c) dynamic multiple lane bi-directional balanced flows (50-50), (d) interspersed bi-directional unbalanced flows (90-10), (c) dynamic multiple lane bi-directional unbalanced flows (90-10).

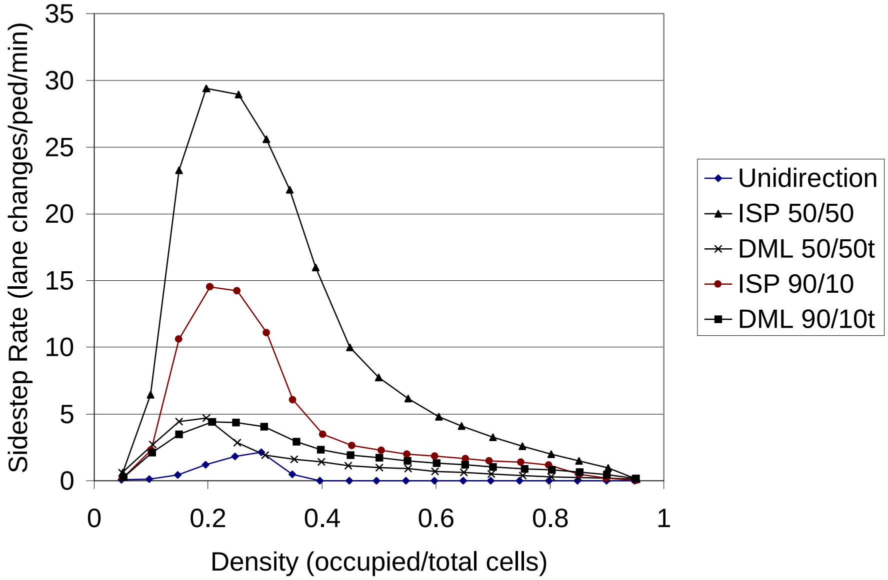

Figure. 5. Sidesteps vs density; p_exchg = 0.5

The results of simulations under conditions of (a) unidirectional flow, (b) interspersed bi-directional balanced flows (50-50), (c) dynamic multiple lane bi-directional balanced flows (50-50), (d) interspersed bi-directional unbalanced flows (90-10), (c) dynamic multiple lane bi-directional unbalanced flows (90-10).

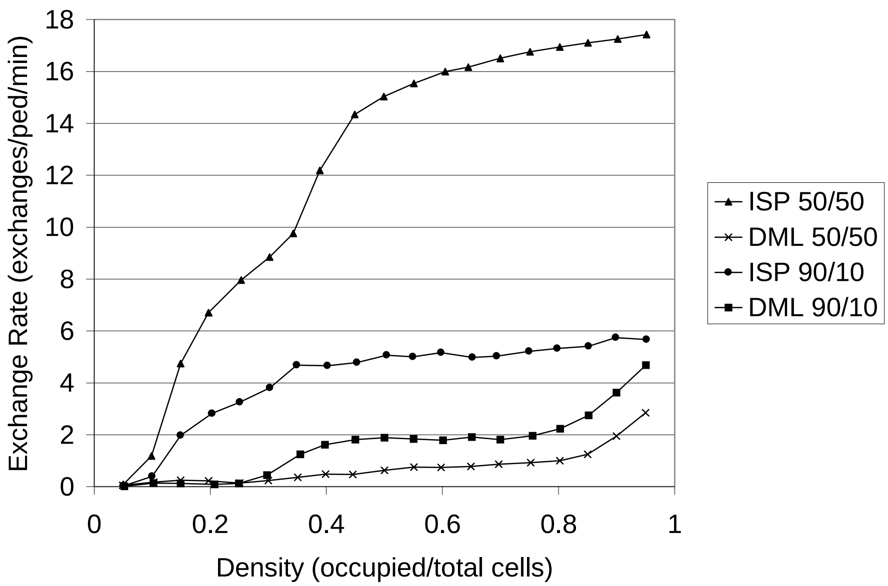

Figure 6. Exchange of place rate vs density; p_exchg = 0.5

The results of simulations under conditions of (a) interspersed bi-directional balanced flows (50-50), (b) dynamic multiple lane bi-directional balanced flows (50-50), (c) interspersed bi-directional unbalanced flows (90-10), (d) dynamic multiple lane bi-directional unbalanced flows (90-10).

As further evidence of the model's capabilities, Figures 3-6 show emergent macroscopic measures of performance over a range of densities for balanced (50-50) and unbalanced (90-10) directional splits and place exchange (*p_exchg)* of 0.5. These figures illustrate the differences between unidirectional, ISP, and DML flows.

Simulations have shown that separated bi-directional flows are essentially equivalent to unidirectional flow (Blue and Adler 1999a, 1999b, 2000). With unidirectional flow and separated flows, very few sidesteps are needed with the restricted lane change rule set used here and generally lane changes are only needed below a density of 0.4 (Figure 5). Restricted lane changing (in two- and three-way ties) in unidirectional flow helps aggregate movement rather than inhibits it. In contrast, when tie-breaking rules are applied that allow some choice in lanes rather than staying in lane, people fall out of step and the aggregate speed drops (Blue and Adler 1999b).

While the fastest and most efficient flow is from unidirectional flow with mode locking, those scenarios that come closest to its conditions do next best. Separated flow is an obvious example. DML 50-50 directional splits fare better than 90-10 (Figures 3-4) because the even directional splits allow better lane formation while the minor direction in the 90-10 split can't form lanes. Evidence of this comes from the 90-10 split needs more exchanges than 50-50. Also, DML 50-50 split has fastest drop-off in sidesteps (Figure 5) and the lowest exchange rate above 0.3 density (Figure 6), indicating excellent lane formation. Conversely, the ISP 90-10 split does better than ISP 50-50, because without rules for lane formation the strong major direction defines de facto lanes.

For the ISP scenarios there is a pronounced cusp in the speed-density curve at 0.3 density for the 90-10 split and at 0.35 for the 50-50 split. The transition at these points from low density to high-density performance is evidently a self-organization effect. Below 1/3 density, the grid is relatively sparsely occupied. Above a density of 1/3 at least one in three cells is occupied, a condition where entities will begin to have more adjacent neighbors than not, restricting sidestepping and avoidance of oncoming walkers. The region centered about 1/3 has the highest drop in speed (steepest slope) for all scenarios.

ISP speed and volume improvement from optimal lane changing is effective only up to 0.3 density after which values are essentially the same. However, sidesteps are reduced by 7 per person per minute at the peak. Exchanges are reduced only at low density.

Two to five exchanges per pedestrian per minute do not seem excessive (Figure 6), especially since they are done in pairs. It is unlikely that ISP 50-50 flow could maintain the high levels of position exchange (more than one every 4 time steps) that a *p_exchg* of 0.5 allows at densities of 0.5 and above (Figure 6). The ISP 50-50 volume curve (Figure 4) levels off at 40 Peds/min/m-of-width that seems unrealistic, but would be an advantage if this type of flow were safe and acceptable. Generally ISP flows are short term, and DML flows or separated flows are prevalent at high density. The model is capable of treating any case.

The Highway Capacity Manual (1994) shows a linear model of speed-density that amounts to a somewhat simplified version of the family of curves shown in Figure 3. Recent work has shown statistical evidence that more closely agree with these CA-based curves (see Blue and Adler, 2000 for a more complete discussion). Though not much empirical data is available on DML and ISP flows, the current edition of the HCM has ascribed to 90-10 flows, where separated lanes do not form, that the peak volumes (see Figure 4) are approximately 85 percent of peak unidirectional or lane separated flow. The bi-directional models show agreement with the HCM in that especially the peak DML volumes (88 percent) and to a lesser extent ISP volumes (77 percent) fall within the HCM's 85 percent range.

DISCUSSION

This bi-directional pedestrian CA model exhibits a range of complex, collective phenomena previously unexplored. This bi-directional model captures formerly intractable flows where discrete automata maneuver at a broad range of densities. The modeled pedestrians appear to exhibit strategic intelligence in choosing the lanes and forward movements with some of the characteristics of actual persons. The self-organization exhibited in the dynamic multiple lane formation (see figure 1) and in the spontaneously arising spatial efficiency of mode locking (see figure 2) reveals that this model exhibits Artificial Life.

While the standard design guide, the Highway Capacity Manual (1994), shows a linear speed-density curve for pedestrians it is generally acknowledged that the unidirectional and separated flow curves would more closely follow the S-curve speed-density relationship realized by the model (see Nagel, 1996 and Blue and Adler, 2000 for discussion). This CA model would further imply that the HCM could indicate that the various types of flow, unidirectional, and bi-directional (a) separated flow, (b) interspersed flow, and (c) dynamic multiple lane flow, have differing speed-flow-density curves.

The model aims at the minimal essential set of rules and parameters to capture bi-directional pedestrian flows. Lane changing and position exchanging are identified as important parameters in modeling ISP and DML flows. Lane change and place exchange variables were set at reasonable values to examine a base case and a mid-range of values. Further calibration from field studies remains to be done.

The CA rules use pedestrian observations of available space and direction of nearby pedestrians. The social forces and gas-kinetic models (see Helbing and Molnar, 1995; Hoogendoorn and Bovy, 2000) imply vectors that give a motivation to act, such as repulsion from others and attractions. In our DML approach persons are repulsed by persons coming from the opposite direction and attracted to following those going in the same direction. However, the

CA model is simpler and closer to emulating what pedestrians actually do: evaluate the space nearby, sidestep others blocking the way (change lanes), move forward as much as possible at a desired speed, and exchange places as needed to avert deadlocking.

Due to the vast amount of walking areas for a growing worldwide population, safe design practices would give this model practical purpose. Facility designers may try to avoid high-density situations, but when high-density pedestrian environments are unavoidable, it is especially important to examine the speed-flow consequences. We are next further studying videotapes of pedestrian flows and expanding the model to four-directional flows (Blue and Adler, 2000) and network flows. Simulations can be observed at http://www.ulster.net/~vjblue.

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